JEE – Advanced 17th May 2026

Paper 02

Question paper and Solution

MATHEMATICS

SECTION 1 (Maximum Marks: 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be equated **according to the following marking scheme**:

  Full Marks : +3 If ONLY the correct option is chosen;
  Zero Marks : 0 if none of the options is chosen (i.e. the question is unanswered);
  Negative Marks : –1 In all other cases.

SECTION 2 (Maximum Marks: 20)

- This section contains **FIVE (05)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be equated **according to the following marking scheme**:

  Full Marks : +4 **ONLY** if (all) the correct option(s) is(are) chosen;
  Partial Marks : +3 If all the four options are correct but **ONLY** three options are chosen;
  Partial Marks : +2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
  Partial Marks : +1 If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
  Zero Marks : 0 if none of the options is chosen (i.e. the question is unanswered);
  Negative Marks : –1 In all other cases.
- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then

  choosing ONLY (A), (B) and (D) will get +4 marks;
  choosing ONLY (A) and (B) will get +2 marks;
  choosing ONLY (A) and (D) will get +2 marks;
  choosing ONLY (B) and (D) will get +2 marks;
  choosing ONLY (A) will get +1 mark;
  choosing ONLY (B) will get +1 mark;
  choosing ONLY (D) will get +1 mark;
  choosing no option (i.e. the question is unanswered) will get 0 marks; and
  choosing any other combination of options will get −1 marks.

SECTION 3 (Maximum Marks: 20)

- This section contains **FIVE (05)** questions.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value corresponding to the answer in the designated place using the mouse and the on-screen virtual numeric keypad.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be equated **according to the following marking scheme**:
  Full Marks : +4 If **ONLY** the correct numerical value is entered in the designated place;
  Zero Marks : 0 In all other cases.

SECTION 4 (Maximum Marks: 8)

- This section contains **TWO (02)** question stems.
- This section contains **TWO (02)** questions corresponding to each question stem.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value corresponding to the answer in the designated place using the mouse and the on-screen virtual numeric keypad.
- If the numerical value has more than two decimal places, truncate/round-off the value to TWO decimal places.
- Answer to each question will be equated **according to the following marking scheme**:
  Full Marks : +2 If **ONLY** the correct numerical value is entered in the designated place;
  Zero Marks : 0 In all other cases.

Section 1
Multiple choice questions with one correct alternative

1. Let $\vec{a}, \vec{b}$ be two vectors, and let P, Q and R be the points with position vectors, $\vec{a}, \vec{b}$ and $\vec{a}+\vec{b}$, respectively, with respect to the origin O. If $|\vec{a}+\vec{b}| = \sqrt{21}, |\vec{a}-\vec{b}| = 3$, and $\vec{a}$ and $(\vec{a}-\vec{b})$ are perpendicular to each other, then the area of the triangle OPR is

(A) $\sqrt{3}$ (B) $\frac{\sqrt{3}}{2}$ (C) $\frac{3\sqrt{3}}{2}$ (D) $\frac{3}{2}$

Ans (C)

$\left|\bar{a}+\bar{b}\right| = \sqrt{21},\ \left|\bar{a}-\bar{b}\right| = 3,\ \bar{a} \perp \bar{a}-\bar{b}$

$\left|\bar{a}+\bar{b}\right|^2 = \left|\bar{a}\right|^2 + \left|\bar{b}\right|^2 + 2\bar{a}\cdot\bar{b} = 21$ …(1)

$\left|\bar{a}-\bar{b}\right|^2 = \left|\bar{a}\right|^2 + \left|\bar{b}\right|^2 - 2\bar{a}\cdot\bar{b} = 9$ …(2)

(1) – (2) $\Rightarrow 4\bar{a}\cdot\bar{b} = 21-9 = 12 \Rightarrow \bar{a}\cdot\bar{b} = 3$

(1) + (2) $\Rightarrow 2\left(\left|\bar{a}\right|^2 + \left|\bar{b}\right|^2\right) = 30 \Rightarrow \left|\bar{a}\right|^2 + \left|\bar{b}\right|^2 = 15$ …(3) $\because \bar{a} \perp \bar{a}-\bar{b}$

$\bar{a}\cdot\left(\bar{a}-\bar{b}\right) = 0 \Rightarrow \left|\bar{a}\right|^2 - \bar{a}\cdot\bar{b} = 0 \Rightarrow \left|\bar{a}\right|^2 = 3$

From (3), $3 + \left|\bar{b}\right|^2 = 15 \Rightarrow \left|\bar{b}\right|^2 = 12$

Vertices of ΔOPQ, $O(\bar{0}), P(\bar{a}), Q(\bar{a}+\bar{b})$

$\text{Area} = \frac{1}{2}\left|\overline{OP}\times\overline{OQ}\right| = \frac{1}{2}\left|\bar{a}\times(\bar{a}+\bar{b})\right|$

$= \frac{1}{2}|\bar{a}\times\bar{a}+\bar{a}\times\bar{b}|$

$= \frac{1}{2}|\bar{a}\times\bar{b}| \qquad (\because \bar{a}\times\bar{a}=\bar{0})$

$= \frac{1}{2}\left(3\sqrt{3}\right)=\frac{3\sqrt{3}}{2}$

$$\left[\begin{aligned} \because |\bar{a}\times\bar{b}|^2 &= |\bar{a}|^2|\bar{b}|^2-(\bar{a}\cdot\bar{b})^2 \\ &= 3\times 12-9 \\ &= 36-9 \Rightarrow 27 \\ |\bar{a}\times\bar{b}| &= 3\sqrt{3} \end{aligned}\right]$$

2. Let T be the tangent to the parabola $y^2 = 16x$ at the point (64, 32). Let be the tangent to the same parabola at another point (x_1, y_1) on the parabola. If L and T are perpendicular to each other, then the distance between the point (x_1, y_1) and the focus of the parabola is

(A) $\frac{15}{4}$ (B) 4 (C) $\frac{17}{4}$ (D) 5

Ans (C)

$y^2 = 4ax$

Given, $y^2 = 16x \Rightarrow 4a = 16 \Rightarrow a = 4$

focus = (4, 0)

Slope of the parabola at (x_0, y_0) is $m=\frac{2a}{y_0}$

Slope at $(64, 32)=\frac{8}{32}=\frac{1}{4}$

The tangent L is perpendicular T

$m_L m_T = -1 \Rightarrow m_L\times\frac{1}{4}=-1 \Rightarrow m_L = -4$

Point of contact $=\left(\frac{a}{m^2}, \frac{2a}{m}\right)=(x_1, y_1)$ = 4, m = –4

$(x_1, y_1)=\left(\frac{1}{4},-2\right)$

Focal distance = $x_1 + a = \frac{1}{4}+4=\frac{17}{4}$

3. Let y : $(-\infty, \infty) \to (0, \infty)$ be the solution of the differential equation $\frac{dy}{dx}=\frac{e^{5x}y^3+y^3}{e^x+e^xy^4}$, satisfying $y(0)=\frac{1}{\sqrt{2}}$. Then the value of $y(\log_e 2)$ is

(A) $\sqrt{\frac{5+\sqrt{35}}{2}}$ (B) $\sqrt{\frac{7+\sqrt{53}}{2}}$ (C) $\frac{7+\sqrt{53}}{2}$ (D) $\frac{5+\sqrt{35}}{2}$

Ans (B)

$\frac{dy}{dx}=\frac{y^3}{e^x}\left[\frac{e^{5x}+1}{1+y^4}\right]$

$$\int \frac{1+y^4}{y^3}dy = \int \frac{e^{5x}+1}{e^x}dx$$

$$\int \left(\frac{1}{y^3}+y\right)dy = \int \left(e^{4x}+e^{-x}\right)dx$$

$$-\frac{1}{2y^2}+\frac{y^2}{2}=\frac{e^{4x}}{4}-e^{-x}+c \quad \because y(0)=\frac{1}{\sqrt{2}}$$

$$-\frac{2}{2}+\frac{1}{4}=\frac{1}{4}-1+c \Rightarrow c=0$$

$$\frac{y^2}{z}-\frac{1}{2y^2}=\frac{e^{4x}}{4}-e^{-x}$$

at $x = \log_e 2$

$$\frac{y^2}{2}-\frac{1}{2y^2}=\frac{e^{4\log_e 2}}{4}-e^{-\log_e 2}$$

$$\frac{y^2}{2}-\frac{1}{2y^2}=4-\frac{1}{2}$$

$$y^2-\frac{1}{y^2}=7$$

$$y^2=a \Rightarrow a-\frac{1}{a}=7$$

$a^2 - 1 = 7a$

$a^2 - 7a - 1 = 0$

$$a=\frac{7\pm\sqrt{49+4}}{2}=\frac{7\pm\sqrt{53}}{2}$$

$$y^2=\frac{7+\sqrt{53}}{2}$$

$$y=\sqrt{\frac{7+\sqrt{53}}{2}}$$

4. The value of the definite integral $\int_0^2 \frac{1}{3^x+3}dx$ is

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{\log_e 3}{3}$ (D) $\frac{\log_e 3}{2}$

Ans (B)

$$\int_0^2 \frac{1}{3^x+3}dx$$

Let $3^x = t$

$\log 3^x = \log t$

$x \log 3 = \log t$

$$dx=\frac{1}{t\log 3}dt$$

$$\int_1^9 \frac{1}{t+3}\cdot\frac{1}{t\log 3}dt = \frac{1}{\log 3}\int_1^9 \frac{1}{t(t+3)}dt$$

$$= \frac{1}{3\log 3}\int_1^9\left(\frac{1}{t}-\frac{1}{t+3}\right)dt$$

$$= \frac{1}{3\log 3}\left[\log t - \text{lot}(t+3)\right]_1^9$$

$$= \frac{1}{3\log 3}\left[\log\frac{t}{t+3}\right]_1^9 = \frac{1}{3\log 3}\left[\log\frac{9}{12}-\log\frac{1}{4}\right]$$

$$= \frac{1}{3\log 3}(\log 3) = \frac{1}{3}$$

Section 2

Multiple choice questions with one or more than correct alternative/s

5. Let R denote the set of all real numbers. Consider the polynomial function $f : R \to R$ defined by $f(x) = \frac{d^{10}}{dx^{10}}\left((x^2-1)^{10}\right)$, for all $x \in R$. Here $\frac{d^{10}}{dx^{10}}\left((x^2-1)^{10}\right)$ is the 10^{th} order derivative of the function $(x^2-1)^{10}$.

Then which of the following statements is (are) TRUE?

(A) The coefficient of x^8 in the polynomial f(x) is $(-10)\left(\frac{18!}{8!}\right)$

(B) The value of f(1) + f(–1) is equal to $10!\ 2^{11}$

(C) The degree of the polynomial f(x) is 10

(D) The constant term of the polynomial f(x) is $-\left(\frac{10!}{5!}\right)$

Ans (A), (B) and (C)

$$f(x) = \frac{d^{10}}{dx^{10}}\left((x^2-1)^{10}\right) = \frac{d^{10}}{dx^{10}}\left({}^{10}C_0x^{20} - {}^{10}C_1x^{18} + {}^{10}C_2x^{16} - {}^{10}C_3x^{14} + {}^{10}C_4x^{12} - {}^{10}C_5x^{10} + ...\right)$$

$$f(x) = {}^{10}C_0 \times \frac{20!}{10!}x^{10} - {}^{10}C_1 \times \frac{18!}{8!}x^8 + ... - {}^{10}C_5 \times \frac{10!}{0!}$$

(A) Coefficient of x^8 in $f(x)$ is $-10\times\frac{18!}{8!}$

(B) $f(1)+f(-1) = 10!(2^{11})$

$$f(x) = \frac{10!}{0!}(x+1)^{10} + {}^{10}C_1\cdot\frac{10!}{1!}(x-1)\times\frac{10!}{1!}(X-1)\times\frac{10!}{9!}(x+1)^9 + {}^{10}C_2\frac{10!}{2!}(x-1)^2\times\frac{10!}{8!}(x+1)^8$$
$$+ ...\ {}^{10}C_{10}\frac{10!}{10!}(x-1)^{10}\times\frac{10!}{0!}$$

$f(1) = 10!(2)^{10}$; $f(-1) = 10!(2)^{10}$

(C) degree of $f(x)$ is 10

(D) constant term in $f(x)$ is $-{}^{10}C_5 \times 10!$.

6. Let a, b, c be positive integers in arithmetic progression such that the equation $ax^2 + bx + c = 0$ has only integer solutions.
Then which of the following statements is (are) TRUE?
(A) c – b is an integer multiple of a
(B) Both the roots of the equation $ax^2+bx+c=0$ are odd integers
(C) If c = 15, then ab = 8
(D) If b = 8, then x = 3 is a root of the equation $ax^2 + bx + c = 0$
Ans (A), (B) and (C)

$ax^2+bx+c=0$; a, b, c $\to$ A.P.

$\alpha+\beta=\frac{-b}{a}$ and $\alpha\beta=\frac{c}{a}$

$b=-a(\alpha+\beta)$ and $c=\alpha\beta$

$2b=a+c\Rightarrow -2a(\alpha+\beta)=a+a\beta\alpha.$

$-2(\alpha+\beta)=1+\alpha\beta\Rightarrow 2\alpha+2\beta+\alpha\beta+1=0$

$(\alpha+2)(\beta+2)=3$

$\alpha=-5$

$\beta=-3$

$\frac{c}{a}=15,\ -1\Rightarrow\frac{b}{a}=8,\ 0$

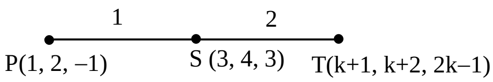
$c=15a$ and $b=8a\Rightarrow c-b=7a$

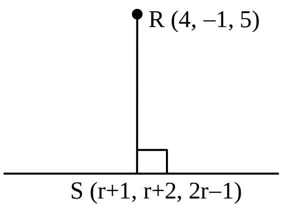
7. Let L be the straight line joining the points P(1, 2, −1) and Q(2, 3, 1). Let S be the foot of the perpendicular drawn from the point R(4, −1, 5) to the line L . Another line passing through R intersects L at a point T such that the point S divides the line segment PT internally in the ratio |PS| : |ST| = 1 : 2 , where |PS| and |ST| are the lengths of the line segments PS and ST, respectively.
Then which of the following statements is (are) TRUE?
(A) The orthocentre of the triangle PRT is $\left(\frac{23}{5},-4,\frac{31}{5}\right)$
(B) The orthocentre of the triangle PRT is (4, 3, 5)
(C) The area of the triangle PRT is $6\sqrt{5}$
(D) The area of the triangle PRT is $18\sqrt{5}$
Ans (A) and (D)

$L:\frac{x-1}{1}=\frac{y-2}{1}=\frac{z+1}{2}$

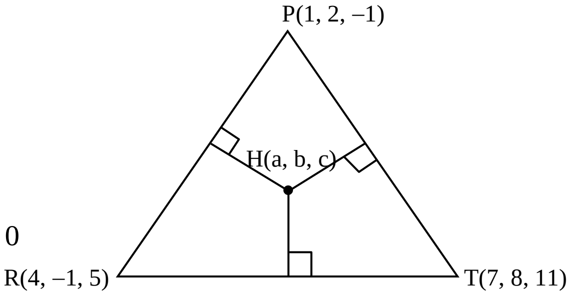
$1(r-3)+1(r+3)+2(2r-6)=0$

$6r=12\Rightarrow r=2\Rightarrow S(3,\ 4,\ 3)$





$3 = \frac{k+1+2}{3} \Rightarrow k = 6 \Rightarrow T(7, 8, 11)$



$\overrightarrow{PR} = 3\hat{i} - 3\hat{j} + 6\hat{k}$

$\overrightarrow{PT} = 6\hat{i} + 6\hat{j} + 12\hat{k} \ \Rightarrow \overrightarrow{PR} \times \overrightarrow{PT} = 18[-4\hat{i} + 2\hat{k}]$

Area $= \frac{18}{2} \times \sqrt{16+4} = 18\sqrt{5}$

For orthocentre $3(a-1) + 9(b-2) + (b-2) + (c+1) = 0$

$\Rightarrow a - 1 + 3b - 6 + 2c + 2 = 0 \Rightarrow a + 3b + 2c = 5$

$\therefore$ Option (A) satisfying.

8. Let y = f(x) be the real valued function defined on the interval (0, ∞), satisfying y(1) = 0 an the differential equation $x\frac{dy}{dx} = y - x^3$.

Then which of the following statements is (are) TRUE?

(A) The function f has a local minimum at $x = \frac{1}{\sqrt{3}}$

(B) The function f has a local maximum at $x = \frac{1}{\sqrt{3}}$

(C) The function f is increasing in the interval (1, 2)

(D) If $g(x) = 4x^3 - 5x^2 + \frac{3}{2}x$ for x > 0, then the number of elements in the set {x ∈ (0, ∞) : f(x) = g(x)} is 2

Ans (B) and (D)

$$x\frac{dy}{dx} = y - x^3 \Rightarrow \frac{dy}{dr} + \frac{y}{-x} = -x^2$$

$$\text{I.F.} = e^{\int \frac{-1}{x}dx} = e^{-\ln x} = \frac{1}{x}$$

$$y\left(\frac{1}{x}\right) = \frac{-x^2}{2} + C \Rightarrow y = \frac{-x^3}{2} + Cx$$

$$y(1) = 0 \Rightarrow 0 = \frac{-1}{2} + C \Rightarrow C = \frac{1}{2}$$

$$f(x) = y = \frac{-x^3}{2} + \frac{x}{2} = \frac{x - x^3}{2}$$

$$f'(x) = \frac{1-3x^2}{2} = 0 \Rightarrow x = \pm\frac{1}{\sqrt{3}}$$

$$f''(x) = -3x \Rightarrow f''\left(\frac{1}{\sqrt{3}}\right) < 0$$

local max, at $x = \frac{1}{\sqrt{3}}$

$f'(x) < 0$ in (1, 2)

$$g(x) = 4x^3 - 5x^2 + \frac{3}{2}x$$

$$4x^3 - 5x^2 + \frac{3}{2}x = \frac{x - x^3}{2}$$

$9x^3 - 10x^2 + 2x = 0 \Rightarrow x = 0 \times x > 0$

$9x^2 – 10x + 2 = 0$

$D > 0$

9. Let R denote the set of all real numbers and let $i = \sqrt{-1}$. Consider the matrices $S = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ and $T = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$. Let a, b, c, d be real numbers such that $ST = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Let H = {x + iy : x, y ∈ R and y > 0}.

Then which of the following statements is (are) TRUE?

(A) $\frac{b+ia}{d+ic} = i$

(B) If $\omega = \frac{-1+i\sqrt{3}}{2}$, then $\frac{a\omega+b}{c\omega+d} = \omega$

(C) If m is an integer greater than 2 such that $(ST)^2 = (ST)^m$, then m is an integer multiple of 8

(D) If z ∈ H, then $\frac{az+b}{cz+d} \in H$

Ans (B), (C) and (D)

$S = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ and $T = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

$ST = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

∴ a = 0, b = –1, c = d = 1

(A) $\frac{b+ia}{d+ic} = \frac{-1}{1+i} = \frac{i-1}{2} \neq i$

(B) $\frac{a\omega+b}{c\omega+d} = \frac{-1}{\omega+1} = \frac{-1}{-\omega^2} = \omega$

(C) $(ST)^2 = (ST)^m$

$(ST)^2 = \begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} -1 & -1 \\ 1 & 0 \end{pmatrix}$

$(ST)^4 = \begin{pmatrix} -1 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} -1 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & -1 \end{pmatrix}$

$(ST)^8 = \begin{pmatrix} 0 & 1 \\ -1 & -1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ -1 & -1 \end{pmatrix} = \begin{pmatrix} -1 & -1 \\ 1 & 0 \end{pmatrix}$

$(ST)^2 = (ST)^m \Rightarrow$ m is an integral multiple of 8

(D) $\frac{az+b}{cz+d} = \frac{-1}{z+1} \in H$

Section 3

Numerical problems (truncate/round-off two decimal places)

10. Let N denote the set of all positive integers. Consider the sets A = {1, 2, 3, 4, 5} and B = {1, 2, 3, 4, 5, 6, 7}. Let S be the set of all functions $f : A \to B$ such that $f(2) \neq 2$ and $f(4) \neq 4$. Consider the set $T = \{f \in S :$ there exists a function $g : B \to N$ such that $g(f(x)) = 2^x$ for all $x \in A\}$.
Then the number of elements in set T is ______________.
Ans 1860.00
$n(A) = 5$, $n(B) = 7$, $g(f(x)) = 2^x$
2^x is increasing function $\Rightarrow$ f(x) must be 1-1 function
$f(2) \neq 2$, $f(4) \neq 4$
Number of 1-1 functions from A to B = $^7p_5 = 7 \times 6 \times 5 \times 4 \times 3$
$= 2520$
Number of 1-1 functions (f(2) = 2) = $^6p_4 = 6 \times 5 \times 4 \times 3 = 360$
Number of 1-1 functions (f(4) = 4) = $^6p_4 = 360$
Number of 1-1 functions (f(2) = 2 and f(4) = 4) = $^5p_3 = 5 \times 4 \times 3$
$= 60$
Number of elements in T = 2520 – (360 + 360 – 60)
= 2520 – 660
= 1860

11. A bookshelf contains 6 distinct books of Mathematics and 5 distinct books of Physics. From these 11 books, 6 books are chosen at random. Let X be the absolute value of the difference between the number of Mathematics books chosen and the number of Physics books chosen. If α is the mean of the random variable X, then the value of 77α is ______________.
Ans 100.00

Maths	Phy	Absolute difference	Value
1	5	4	$^6C_1 \times {}^5C_5 = 6$
2	4	2	$^6C_2 \times {}^5C_4 = 75$
3	3	0	$^6C_3 \times {}^5C_3 = 200$
4	2	2	$^6C_4 \times {}^5C_2 = 150$
5	1	4	$^6C_5 \times {}^5C_1 = 30$
6	0	6	$^6C_6 \times {}^5C_0 = 1$

Total number of ways chosen 6 from 11 = $^{11}C_6 = 462$

$$\alpha = \text{Mean}(\bar{x}) = \frac{4\times6+2\times75+0\times200+2\times150+4\times30+6\times1}{462}$$

$$= \frac{600}{462} = \frac{100}{77} \Rightarrow 77\alpha = 100$$

12. Consider a data consisting of 10 observations $x_1, x_2, \ldots, x_{10}$, whose mean is 5 and variance is 7. If the mean and the variance of the first 8 observations $x_1, x_2, \ldots, x_8$ are 4 and 3.5, respectively and $x_9 < x_{10}$, then the value of $3x_9 + 2x_{10}$ is _____________.
Ans 44.00

$\sum_{i=1}^{10} x_i = 50;\ \bar{x} = 5,\ \sigma^2 = 7,\ n = 10$

$\sigma^2 = \frac{\sum_{i=1}^{10} x_i^2}{n} - (\bar{x})^2$

$\sum_{i=1}^{10} x_i^2 = 320$

$\sum_{i=1}^{8} n_{1i} = 32,\ \sigma_1^2 = 3.5,\ n_1 = 8,\ \bar{x}_1 = 4$

$\sigma_1^2 = \frac{\sum_{i=1}^{8} x_{1i}^2}{n_1} - (\bar{x}_1)^2$

$\sum x_{1i}^2 = 156$

$x_9 + x_{10} = \sum_{i=1}^{10} x_i - \sum_{i=1}^{8} x_{1i} = 50 - 32 = 18$

$x_9^2 + x_{10}^2 = \sum_{i=1}^{10} x_i^2 - \sum_{i=1}^{8} x_{1i}^2 = 320 - 156 = 164$

$(x_9 + x_{10})^2 = (x_9^2 + x_{10}^2) + 2x_9x_{10}$

$x_9x_{10} = 80$ and $x_9 + x_{10} = 18$

$x_9 = 8$ and $x_{10} = 10$

$3(x_9) + 2(x_{10}) = 44$

13. Consider the ellipse E given by $\frac{x^2}{18} + \frac{y^2}{12} = 1$. Let H be the hyperbola whose eccentricity is the reciprocal of the eccentricity of E and whose foci are the same as that of E. Let P and Q be the points of intersection of H and the parabola $\sqrt{5}\,y = x^2$ in the first quadrant. Let d be the distance between P and Q.
If a and b are the integers such that $d^2 = a + b\sqrt{5}$, then the value of a – b is __________.

Ans 18.00

$E : \frac{x^2}{18} + \frac{y^2}{12} = 1$

$e_H = \frac{1}{e_E}$. $F_E(ae, 0) = f_4(ae, 0)$

$a^2 = 18,\ b^2 = 12$

$e^2E = 1 - \frac{b^2}{a^2} \Rightarrow e_E^2 = \frac{1}{3} \Rightarrow e_E = \frac{1}{\sqrt{3}}$

$e_H = \sqrt{3}$

$f_E(ae, 0) = (\pm\sqrt{6}, 0)$

$f_H(\pm\sqrt{6}, 0)$

$a_He_H = \pm\sqrt{6}$

$e_H = \sqrt{3} \Rightarrow a_H = \pm\sqrt{2}$

$b_H^2 = a_H^2(e_H^2 - 1)$

$b_H^2 = 4$

$H: \frac{x^2}{2} - \frac{y^2}{4} = 1 \Rightarrow 2x^2 - y^2 = 4 \quad \ldots(1)$

Given $x^2 = \sqrt{5}y \ \ldots(2)$

Solve (1) and (2) we get, $P\left(\sqrt{5+\sqrt{5}}, \sqrt{5}+1\right), Q\left(\sqrt{5-\sqrt{5}}, \sqrt{5}-1\right)$

$d^2 = 14 - 4\sqrt{5} \Rightarrow a = 14, b = -4$

a – b = 18

14. For a real number α, let [α] denote the greatest integer less than or equal to α. For a finite set S, let |S| denote the number of elements in the set S.
Consider the functions $f : (-3, 3) \to (-\infty, \infty)$ and $g : (-3, 3) \to (-\infty, \infty)$ defined by
$f(x) = [x^3] \log_e(1 + \sin^2(\pi(x - [x])))$ and $g(x) = x^3 \sin^2(\pi \log_e(1 + x - [x]))$.
Let $A = \{x \in (-3, 3) : f \text{ is discontinuous at } x\}$ and $B = \{x \in (-3, 3) : g \text{ is discontinuous at } x\}$.
Then the value of $|A| + 2|B| - |A \cap B|$ is ____________.

Ans 56.00

$f(x) = [x^3] \log_e(1 + \sin^2(\pi(x - [x])))$

$\forall x \in Z; \log(1 + \sin^2(\pi\{x\})) = 0$ and it is continuous at $\{-2, -1, 0, 1, 2\}$

f(x) is discontinuous where x^3 is integer

As $x \in (-3, 3) \Rightarrow x^3 \in (-27, 27)$

Number of integral values are $27 \times 2 - 1 = 53$

A = 53 – 5 = 48

Now, $g(x) = x^3 \sin^2(\pi \log_e(1 + x - [x]))$

As $x = n \in Z$

If $x \to n^+$ $\{x\} \to 0$

$g(n^+) = n^3 \sin^2(\pi(\log(1)) = 0$

If $x \to n^-$ $\{x\} \to 1$

$g(n^-) = n^3 \sin^2(\pi(\log(2))$, if n = 0 then $g(n^-) = 0$

g(x) is discontinuous x = {–2, –1, 1, 2}

B = 4 and $(A \cap B) = 0$

$|A| + 2|B| - |A \cap B| = 56$

Stem – 1

Question Stem for Question number 15 and 16

Consider the curve C_1 given by $y = e^x$ for $x \in [0, 10\pi]$, and the curve C_2 given by $y = e^{-x}(\sin x + \cos x)$ for $x \in [0, 10\pi]$.

Let n be the total number of points of intersection of the curves C_1 and C_2.

Suppose that $\alpha_1, \alpha_2, \ldots, \alpha_n \in [0, 10\pi]$ are the x-coordinates of the point of intersection of the curves C_1 and C_2 such that $\alpha_1 < \alpha_2 < \ldots < \alpha_n$

15. The value of n is ____________.

Ans 11.00

$e^{-x} = e^{-x}(\sin x + \cos x)$

$e^{-x}(\sin x + \cos x - 1) = 0$

$e^{-x}=0$ or $\sin x+\cos x=1$

$\sqrt{2}\left(\sin\left(x+\frac{\pi}{4}\right)\right)=1$

$\sin\left(x+\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}}$

Number of solutions is 11

16. Let β be the area of the region enclosed between the curves C_1, C_2 and the lines $x = \alpha_1$ and $x = \alpha_4$. Then the value of $-\frac{1}{\pi}\log_e\left(\beta-2\,e^{-\frac{\pi}{2}}\right)$ is ___________.

Ans 2.50

$$\beta=\left|\int_0^{\frac{\pi}{2}}\left(e^{-x}-e^{-x}\left(\sin x+\cos x\right)\right)dx\right|+\left|\int_{\frac{\pi}{2}}^{2\pi}\left(e^{-x}-e^{-x}\left(\sin x+\operatorname{cox}\right)\right)dx\right|+\left|\int_{2\pi}^{\frac{5\pi}{2}}\left(e^{-x}-e^{-x}\left(\sin x+\operatorname{cox}\right)\right)dx\right|$$

$$\beta=\left|\int_0^{\frac{\pi}{2}}e^{-x}\left(1-\cos x\right)dx\right|+\left|\int_{\frac{\pi}{2}}^{2\pi}e^{-x}\left(1-\cos x\right)dx\right|+\left|\int_{2\pi}^{\frac{5\pi}{2}}e^{-x}\left(1-\cos x\right)dx\right|$$

$$\beta=e^{-\frac{5\pi}{2}}+2e^{-\frac{\pi}{2}}$$

Now, $-\frac{1}{\pi}\ln\left(\beta-2e^{-\frac{\pi}{2}}\right)=-\frac{1}{\pi}\ln\left(e^{-\frac{5\pi}{2}}\right)$

$$=-\frac{1}{\pi}\times\frac{-5\pi}{2}=\frac{5}{2}=2.5$$

Stem - 2

Question Stem for Question number 17 and 18

Consider the ellipse given by $x^2 + 4y^2 = 1$ and $4x^2 + y^2 = 1$.

17. Let P be the point in the first quadrant where the given ellipses intersect. If θ is the acute angle between the tangents to the given ellipses at the point P, then the value of 4 tan θ is ___________.

Ans: 7.50

$x^2 + 4y^2 = 1$ …(1)

$4x^2 + y^2 = 1$ …(2)

Solve (1) and (2) $x=\frac{1}{\sqrt{5}}\,y=\frac{1}{\sqrt{5}}$

$P=\left(\frac{1}{\sqrt{5}},\frac{1}{\sqrt{5}}\right)$

From (1), Slope of tangent at P

$m_1=\frac{-x}{4y}=\frac{-1}{4}$

From (2), Slope of tangent at P

$m_2=\frac{-4x}{y}=-4$

$$\text{Tan}\,\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\frac{-\frac{1}{4} + 4}{1 + 1}\right| = \frac{15}{8}$$

$$4\,\text{Tan}\,\theta = \frac{15}{2}$$

18. If α is the area of the common region that lies inside both the given ellipses, then the value of cot α is ____________.

Ans 0.75

$$x^2 + 4y^2 = 1 \Rightarrow r^2\left(\cos^2\phi + 4\sin^2\phi\right) = 1 \Rightarrow r^2 = \frac{1}{1 + 3\sin^2\phi}$$

$$4x^2 + y^2 = 1 \Rightarrow r^2\left(4\cos^2\phi + \sin^2\phi\right) = 1 \Rightarrow r^2 = \frac{1}{1 + 3\cos^2\phi}$$

$$\text{Area}\left(Q_1\right) = 2\int_0^{\frac{\pi}{4}} \frac{1}{2} r^2 = \int_0^{\frac{\pi}{4}} \frac{1}{1 + 3\cos^2\phi}\, d\phi = \int_0^{\frac{\pi}{4}} \frac{\sec^2\phi}{\text{Tan}^2\phi + 4}\, d\phi = \frac{1}{2}\text{Tan}^{-1}\frac{1}{2}$$

$$\text{Area} = 4 \times \frac{1}{2}\text{Tan}^{-1}\frac{1}{2} = 2\,\text{Tan}^{-1}\frac{1}{2}\ (\text{or})\ \text{Tan}^{-1}\frac{4}{3}$$

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